Appendix A

Proof of Proposition 1

The Church can commit to punishing anyone who borrows more than is necessary to reach subsistence when young (by denying them a transfer when old) only if W is small enough that the Church maximizes its utility by not ensuring subsistence to *all* agents. The Church can punish "over-borrowers" either by providing transfers to other agents or keeping what would have been the agent's transfer for itself (when the Church's marginal return to keeping the last agent free (MRF) equals its marginal return to consumption (MRC)). Employing this logic, I determine the Church's optimal transfer scheme when it bans interest and when it allows interest when W is small enough that it does not transfer to all agents, treating each case separately, and comparing the optima to derive a global optimum. I then determine the value of W (denoted W^*) in which the Church does not provide transfers to all when $W \leq W^*$.

If the Church only provides transfers to keep agents at subsistence, when $p_t = A$ the optimal transfer scheme to an agent who receives a transfer is:

If $\mu(1+r) \leq 1$,

$$\left\{ T_{y,t}^i = 0;\ T_{o,t}^i = \begin{cases} \Upsilon & \text{if } b^i = m - w_y, w_o = w_o^L \\ 0 & \text{otherwise} \end{cases} \right\} \tag{A1}$$

If $\mu(1+r) > 1$,

$$\left\{ T_{y,t}^i = m - w_y;\ T_{o,t}^i = \begin{cases} m - w_o^L & \text{if } b^i = 0, w_o = w_o^L \\ 0 & \text{otherwise} \end{cases} \right\} \tag{A1'}$$

where $\Upsilon = m - w_o^L + (1+r)(m - w_y)$. On the other hand, when interest is banned the optimal (long-run) transfer scheme to an agent who receives a transfer is:

$$\left\{ T_{y,t}^i = m - w_y;\ T_{o,t}^i = \begin{cases} m - w_o^L & \text{if } b^i = 0, w_o = w_o^L \\ 0 & \text{otherwise} \end{cases} \right\} \tag{A2}$$

The Church optimizes by choosing the p_t which minimizes the transfer per agent – this entails that the long-run population is maximized at the lowest cost. When $p_t = B$, the per agent transfer is $\Gamma = m - w_y + \mu(m - w_o^L)$. This is the same per agent transfer when $p_t = A$ and $\mu(1+r) > 1$, so in this case the Church allows interest (since it breaks indifference by choosing $p_t = p_{t-1}$). On the other hand, when $\mu(1+r) \leq 1$, $p_t = A$ entails that the per agent transfer is $\mu\Upsilon$. The Church never bans interest, since $\Gamma > \mu\Upsilon$ when $\mu(1+r) \leq 1$.

This analysis only holds when the Church does not transfer to all agents – that is, when W is small enough such that MRF<MRC when the Church provides subsistence for all. Otherwise, (A1) and (A1') are not part of a subgame perfect Nash equilibrium, as the Church cannot commit to not providing a greater transfer to agents who borrow more than is necessary to reach subsistence and incur a negative income shock. W^* is implicitly defined by equations (A3) and (A4):[1]

If $\mu(1+r) \leq 1$,

$$1 = v(W^* - \mu(N-1)\Upsilon) - v(W^* - \mu N\Upsilon) \quad \text{(A3)}$$

If $\mu(1+r) > 1$,

$$1 = v\left(W^* - (N-1)\left(m - w_y + \mu(m - w_o^L)\right)\right) - v\left(W^* - N\left(m - w_y + \mu(m - w_o^L)\right)\right) \quad \text{(A4)}$$

Proof of Proposition 2

When W is large enough that the Church maximizes its utility by ensuring subsistence for all agents, the Church cannot commit to punishing an agent who borrows more than is necessary to reach subsistence. In this case, if the Church provides just enough to keep all agents at subsistence, an agent can borrow a little more and the Church cannot commit to punishing him, since MRF>MRC. Using this logic, I determine the Church's optimal transfer scheme when it bans interest and when it allows interest if $W > W^*$, treating each case separately, and comparing the optima to derive a global optimum.

When $p_t = B$, the Church's optimal transfer scheme to agent *i* is:

$$\left\{\begin{array}{c} T_{y,t}^i = \left\{\begin{matrix} \Phi \\ m - w_y \end{matrix} \text{ if } \begin{matrix} \Omega_B^1 < \Omega_B^2 \\ \text{otherwise} \end{matrix}\right\}; \\ T_{o,t}^i = \left\{\begin{matrix} m - w_o^L \\ 0 \end{matrix} \text{ if } \begin{matrix} w_o = w_o^L, b^i \in \{0, b^{oo}, b^*(\Phi)\} \\ \text{otherwise} \end{matrix}\right\} + \left\{\begin{matrix} (1+r)b^*(\Phi) \\ (1+r)b^{oo} \\ 0 \end{matrix} \text{ if } \begin{matrix} b^i = b^*(\Phi), w_o = w_o^L, \Omega_B^1 < \Omega_B^2 \\ b^i = b^{oo}, w_o = w_o^L, \Omega_B^1 > \Omega_B^2 \\ \text{otherwise} \end{matrix}\right\} \end{array}\right\} \text{(A5)}$$

where $b^*(x)$ is the level of borrowing (at interest) that maximizes an agent's utility when it receives transfer x

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[1] When $W > W^*$, the assumption $T_{y,t}^* + T_{o,t}^* < W$ implies that the Church can afford to provide subsistence to all agents. Moreover, MRF>MRC if and only if the Church's utility is greater when it provides a transfer to the "last agent" when young and old than it is without the agent in the economy. Even if this agent borrows and receives $w_o = w_o^H$, the Church would merely face the same problem in the following period. Since $\mu > 0$ (in some period, the last agent would receive $w_o = w_o^L$) and $\beta \to 1$, $W > W^*$ is a necessary and sufficient condition for MRF>MRC over an infinite horizon.

(given that it is assured subsistence if $w_o = w_o^L$). b^{oo} is the level of borrowing in which MRF=MRC if all non-pious agents choose $b^i = b^{oo}$. Also,

$$T^* = \text{argmin}_{T\geq 0}\,\Psi(T) = T + \mu(1+r)b^*(T) \tag{A6}$$

$$\Phi = \max\{m - w_y, T^*\} \tag{A6'}$$

$$\Omega_B^1 = \Phi + \mu(1-\alpha)(1+r)b^*(\Phi) \tag{A6''}$$

$$\Omega_B^2 = m - w_y + \mu(1-\alpha)(1+r)b^{oo} \tag{A6'''}$$

In words, (A5) states that the Church transfers to attain the least expensive of the following: 1) give agents $\Phi > 0$ when young so they will borrow less to reach their argmax; 2) give young agents zero transfers beyond those necessary to keep all at subsistence, with non-pious agents borrowing until MRF=MRC. Non-pious agents borrow $b^i = \min\{b^*(\Phi), b^{oo}\}$, and the Church's mean transfer to each agent is:

$$\Gamma_B = \mu(m - w_o^L) + \min\{\Omega_B^1, \Omega_B^2\} \tag{A7}$$

On the other hand, when the Church optimizes by transferring to all agents, its optimal transfer scheme to agent *i* when $p_t = A$ is:

$$\left\{\begin{matrix} T_{y,t}^i = \left\{\begin{matrix} T^* \\ 0 \end{matrix} \text{ if } \begin{matrix} \Omega_A = \Psi(T^*) \\ \text{otherwise} \end{matrix}\right\}; \\ \\ T_{o,t}^i = \left\{\begin{matrix} (1+r)b^*(T^*) + m - w_o^L \\ (1+r)b^o + m - w_o^L \\ 0 \end{matrix} \text{ if } \begin{matrix} b^i = b^*(T^*), w_o = w_o^L, \Omega_A = \Psi(T^*) \\ b^i = b^o, w_o = w_o^L, \Omega_A = \mu(1+r)b^o \\ \text{otherwise} \end{matrix}\right\} \end{matrix}\right\} \tag{A8}$$

Where b^o is the level of borrowing in which MRF=MRC if all agents choose $b^i = b^o$, and:

$$\Omega_A = \min\{\Psi(T^*), \mu(1+r)b^o\} \tag{A9}$$

Agents borrow $b^i = \min\{b^*(T^*), b^o\}$, and the Church's transfer to each agent is:

$$\Gamma_A = \mu(m - w_o^L) + \Omega_A \tag{A10}$$

Thus, when $W > W^*$, the Church bans interest when $\Gamma_B < \Gamma_A$, or

$$\min\{\Omega_B^1, \Omega_B^2\} < \Omega_A \tag{A11}$$

From (A11) there are three cases when $W > W^*$ and α is sufficiently large, depending on the value of W:

1) $\Omega_B^1 > \Omega_B^2$ and $\Psi(T^*) > \mu(1+r)b^o$: In this case, MRF=MRC whether interest is banned or allowed. Since there are less agents borrowing at interest when $p_t = B$, b^{oo} increases more than b^o with a marginal increase in W. But this also entails that the Church's threat of punishment is more credible, so the level of *total* transfers increases by less when the Church bans interest than when it does not.

Hence, the parameter set over which interest is banned is increasing in W. Moreover, the parameter set over which interest is banned is increasing in α, as Ω_B^2 is decreasing in α while $\mu(1+r)b^o$ is constant in α. This case holds as W increases until the following case arises.

2) $\Omega_B^1 < \Omega_B^2$ and $\Psi(T^*) > \mu(1+r)b^o$: In this case, MRF=MRC when interest is allowed but agents borrow at their maxima when interest is banned. This is where the clause "if α is sufficiently large" has bite: if α is large enough, per agent borrowing is greater when interest is banned, so agents reach their maxima at a lower level of W than they do when interest is allowed. Ω_B^1 is decreasing in α but constant in W while $\mu(1+r)b^o$ is increasing in W but constant in α, so the parameter set over which interest is banned is increasing in W and α. This case holds until the following case arises.

3) $\Omega_B^1 < \Omega_B^2$ and $\Psi(T^*) < \mu(1+r)b^o$: In this case, agents borrow at their maxima when interest is banned or allowed. Once W is sufficiently large such that this case arises, this case holds for any increase in W. Ω_B^1 is decreasing in α and $\Psi(T^*)$ is constant in α, so the range over which interest is banned is increasing in α. Both Ω_B^1 and $\Psi(T^*)$ are constant in W, so interest will either be banned or allowed for all Win this range (note that if it is allowed in this case, it is allowed over all W). Hence all that is necessary to complete the proof is to show that some parameter set exists in which interest is banned in this case. Indeed, when μ is large enough that $\Psi(T^*) > m - w_y$, which is definitely true when $\mu(1+r) > 1$, there must exist some α for which interest is banned (to see this, note that $\Omega_B^1 \approx m - w_y$ when $\alpha \longrightarrow 1$ and $\Phi = m - w_y$, and $\Omega_B^1 < \Psi(T^*)$ ∀ α when $\Phi = T^*$).

From the above logic, it follows that when α is sufficiently large, the parameter space over which interest is banned in equilibrium is weakly increasing in W and α.